Math 10B with Professor Stankova
Quiz 8; Tuesday, 3/19/2019
Section #206; Time: 9:30 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** $\bar{X} = \mu$ because both are equal to the average value.

> **Solution:** $\bar{X}$ is a random variable that represents an average of a sample (a section's average) where as $\bar{\mu}$ represents the population's average.

2. True **FALSE** The CLT tells us that $\bar{X}$ is normally distributed.

> **Solution:** The CLT tells us that $\bar{X}$ is **approximately** normally distributed.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) Suppose that each student has a 10% chance of going to office hours and this probability is independent of whether other students go.

   (a) (2 points) Choose a random student. Let X be the random variable that outputs 1 if they goes to office hours and 0 otherwise. What is $E[X]$ and $SE(X)$? (Simplify your answer)

   > **Solution:** This is a Bernoulli trial with probability of success $p = 0.1$. Then $E[X] = p = 0.1$ and $SE(X) = \sqrt{Var(X)} = \sqrt{p(1-p)} = \sqrt{0.1(0.9)} = 0.3$.

   (b) (4 points) What is the probability that in a section of 25 students, at most $4\% (= \frac{1}{25})$ of them go to office hours? (You do not need to simplify your answer)

   > **Solution:** This is repeating 25 Bernoulli trials so this is a binomial distribution. We have $n = 25, p = 0.1$ and the probability that at most 1 goes to office hours is
   > $$f(0) + f(1) = \binom{25}{0}(0.1)^0(0.9)^{25} + \binom{25}{1}(0.1)^1(0.9)^{24}.$$

(c) (4 points) Use the CLT to approximate the probability that at most 4% of the section of 25 students go to office hours. (Hint: $z(1) = 0.3413$)

Solution: Let $\bar{X}$ be the average of the X_i for the students, which is the percentage of students who go to office hours in a section of $n = 25$ students. Then $\bar{\mu} = \mu = 0.1$ and $\bar{\sigma} = \sigma/\sqrt{n} = 0.3/\sqrt{25} = 0.06$. We want to calculate $P(\bar{X} \leq 0.04)$. The CLT tells us that $\bar{X}$ is approximately normally distributed so using z scores, this probability is approximately $1/2 - z(|0.04 - 0.1|/0.06) = 1/2 - z(1) = 0.1387$.